

October 9, 2017
Time : 55 minutes
Fall 2017-18

MATHEMATICS 218
QUIZ I

NAME Key
ID# -----

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12
2 W	1 W	11 W	2 M	1 M	11 M	4 M	3 M	2 Th	11 F	4 F	5 F

PROBLEM GRADE

PART I

1 ----- / 14

2 ----- / 13

3 ----- / 9

4 ----- / 15

PART II

5	6	7	8	9	10	11
a	a	a	a	a	a	a
b	b	b	b	b	(b)	b
c	(c)	c	(c)	(c)	c	c
d	(d)	d	(d)	d	d	(d)
e	e	e	e	e	e	e

5-11 ----- / 28

PART III

12	13	14	15	16	17	18
(T)	T	T	(T)	(T)	(T)	T
F	(F)	(F)	F	F	F	(F)

12-18 ----- / 21

TOTAL ----- / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of a and b for which the following system

$$\begin{aligned}x + y + z &= b \\ -3x - 3y + az &= 8 \\ 2x + y + z &= 1\end{aligned}$$

has

- a. no solution
- b. a unique solution
- c. infinitely many solutions.

[14 points]

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b \\ -3 & -3 & a & 8 \\ 2 & 1 & 1 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} 3R_1 + R_2 \\ -2R_1 + R_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & 0 & a+3 & 3b+8 \\ 0 & -1 & -1 & -2b+1 \end{array} \right] \xrightarrow{\begin{array}{l} \text{interchange} \\ R_2 \leftrightarrow R_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & -1 & -1 & -2b+1 \\ 0 & 0 & a+3 & 3b+8 \end{array} \right]$$

a) No Solution

$$\begin{aligned}a+3 &= 0 \text{ and } 3b+8 \neq 0 \\ \therefore a &= -3 \quad \times \quad b \neq -\frac{8}{3}\end{aligned}$$

b) Unique Solution

$$\begin{cases} a+3 \neq 0 \\ a \neq -3 \end{cases}$$

c) Infinitely many solutions

$$\begin{aligned}\text{if } a+3 &= 0 \quad \times \quad 3b+8 = 0 \\ \text{i.e. } a &= -3 \quad \times \quad b = -\frac{8}{3}\end{aligned}$$

2. Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

(a) Find $(A^{-1} + I)^{-1}$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array} \right] \quad [8 \text{ points}]$$

$\underbrace{I}_{A^{-1}}$

$$A^{-1} + I = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \begin{matrix} 2 \text{ identical rows} \\ \det = 0 \\ \text{not invertible} \end{matrix}$$

or

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \quad A^{-1} + I \text{ is } \underline{\text{not}} \text{ invertible}$$

Find $(A^{-1} + 2I)^{-1}$

$$A^{-1} + 2I = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{-2R_1+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 0 & -2 \end{array} \right)$$

$$\xrightarrow{R_3 \cdot \frac{1}{3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & \frac{2}{3} \end{array} \right) \xrightarrow{-2R_3+R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & \frac{4}{3} \end{array} \right)$$

(b) Find A^{11}

$$A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$\text{So } A^{11} = A^{10} \cdot A = (A^2)^5 \cdot A = I^5 \cdot A = A$$

$$\begin{aligned} (A^{-1} + 2I)^{-1} &= \left(\begin{array}{ccc} \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{3} & 0 & \frac{4}{3} \end{array} \right) \\ &= \left(\begin{array}{ccc} \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{3} & 0 & \frac{4}{3} \end{array} \right) \end{aligned}$$

3. Show that for all real numbers a , b and c , the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ can be written as a linear combination of the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad [9 \text{ points}]$$

$$\Rightarrow \begin{cases} c_1 + c_2 + c_3 = a \\ c_1 + 2c_2 + 2c_3 = b \\ 3c_3 = c \end{cases}$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & a \\ 1 & 2 & 2 & b \\ 0 & 0 & 3 & c \end{array} \right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|cc} 1 & 1 & 1 & a \\ 0 & 1 & 1 & b-a \\ 0 & 0 & 3 & c \end{array} \right]$$

\Rightarrow The above system has a unique solution
for any choice of a, b, c

4. Let A and B be 2×2 matrices such that $AB+2A = -I$.

(a) Find an expression of A^{-1} in terms of B

[8 points]

$$A(B + 2I) = -I$$

$$\text{So } A^{-1} = \frac{B + 2I}{-1} = -B - 2I$$

(b) Suppose that the above matrices A and B are given to be $A = B = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$ for some

nonzero number a. Find all possible values of a. $a \neq 0$

[7 points]

$$A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \Rightarrow \tilde{A}^{-1} = \frac{1}{a^2} \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \\ = \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{a} \end{pmatrix}$$

$$\text{Also } \tilde{A}^{-1} = -B - 2I = \begin{pmatrix} -a & 0 \\ 0 & -a \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -a-2 & 0 \\ 0 & -a-2 \end{pmatrix}$$

$$\text{So } -a-2 = \frac{1}{a}$$

$$\Rightarrow -a^2 - 2a = 1$$

$$\Rightarrow a^2 + 2a + 1 = 0$$

$$\Rightarrow (a+1)^2 = 0$$

$$\Rightarrow a = -1$$

PART II. Circle the correct answer for each of the following multiple choice problems (Problem 5 to Problem 11) IN THE TABLE IN THE FRONT PAGE. [4 points for each correct answer].

5. Let A be a 3×3 matrix such that $A^2 = A$. Then,

- a. A is invertible
- b. $\det(A) = 0$
- c. $A = I$
- d. $A^5 = A^2$
- e. None of the above.

$$\begin{aligned} A^2 = A &\Rightarrow A^3 = A^2 = A \\ &\Rightarrow A^4 = A^2 = A \\ &\Rightarrow A^5 = A^2 \end{aligned}$$

[4 points]

6.

If A and B are invertible 3×3 matrices such that

$$\det(2A^{-1}) = 2 = \det(A^3(B^{-1})')$$

Then,

- a. $\det(B) = 1/2$
- b. $\det(B) = 2$
- c. $\det(B) = 32$
- d. $\det(B) = 8$
- e. None of the above.

$$\begin{aligned} \det(2A^{-1}) = 2 \cdot |A^{-1}| &= \frac{8}{|A|} = 2 \Rightarrow |A| = 4 \\ \det(A^3 \cdot (B^{-1})^T) &= |A|^3 \cdot |B^{-1}| = \frac{4^3}{|B|} = 2 \\ &\Rightarrow |B| = 32 \end{aligned}$$

[4 points]

7. Which one of the following statements is **TRUE**?

- a. If $AB = AC$, then $B = C$.
- b. If $AB = 0$ then $A = 0$ or $B = 0$.
- c. If A is invertible then $Ax = x$ has only the trivial solution for x .
- d. If b can be written as a linear combination of the columns of A then $AX = b$ is consistent.

[4 points]

8. Let A be an invertible $n \times n$ matrix. Which one of the following statements is FALSE:

- (a) A^t is invertible.
- (b) The number of nonzero rows in a row echelon form of A is n .
- (c) AB is invertible for any $n \times n$ matrix B .
- (d) $\det(A) \neq 0$.
- (e) The reduced row echelon form of A is I .

[4 points]

